12th International Scientific Conference

BUSINESS AND MANAGEMENT 2022

May 12–13, 2022, Vilnius, Lithuania

ISSN 2029-4441 / eISSN 2029-929X
ISBN 978-609-476-288-8 / eISBN 978-609-476-289-5
Article Number: bm.2022.762
https://doi.org/10.3846/bm.2022.762

FINANCE AND INVESTMENT: NEW CHALLENGES AND OPPORTUNITIES *http://vilniustech.lt/bm*

IMPROVING THE QUANTIFICATION OF INTEREST RATE RISK

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Received 7 March 2022; accepted 7 April 2022

Abstract. The value of Macaulay duration, probably the most widely used quantification method for measuring interest rate sensitivity of bonds, could roughly be financially interpreted as a percentage change of the bond price if the parallel shift of the interest rate equals 1 percentage point along the entire zero-coupon curve and the initial bond price is equal to 100%.

The main problem of its practical application lies in the fact that parallel curve shift is a very rare case, and we are more often concerned with predicting short-term rate shifts and considering their consequences for the rest of the yield curve and thus also for bonds with longer maturities. Therefore, it is useful to find a certain value that represents a quantification of the impact of short rate shifts on bond prices with respect to the parameters of bonds.

So, the main contribution of this financial engineering research is to design a measure that can be used in the same way as Macaulay duration, but as a response to the change of the short interest rate, for example: in the equation for changing ΔP of a bond, in the equation of the volatility ratio of two bonds, or in the equation for bond portfolio sensitivity. Such a measure is still lacking in finance. We refer to this measure as the "short rate-shift duration". Since the effect of the short rate shift on the entire yield curve, and thus especially on the price of long-term bonds, is very difficult to predict analytically, we use empirical data to calculate the duration value of the short-term shift and also to calculate its values for the USD and EUR interest markets.

Keywords: Short rate shift duration, Macaulay duration, interest rate sensitivity, zero-coupon yield curve.

JEL Codes: G1, G12.

Introduction

The value of Macaulay duration, probably the most widely used quantification method for measuring interest rate sensitivity of bonds, could roughly be financially interpreted as a percentage change of the bond price if the parallel shift of the interest rate equals 1 percentage point along the entire zero-coupon curve and the initial bond price is equal to 100%, where the size of this parallel interest rate shift is actually equal to the change of Yield To Maturity (YTM) of this bond. Based on this, we can, for example, conclude that in certain cases the ratio of the average price changes of two bonds should be approximately the same as the ratio of their Macaulay durations.

However, the use of the Macaulay duration value has little practical significance for estimating a change of the price of a bond if this change is due to a shift of the short interest rate, which is the most common case in financial practice compared to a parallel shift of the entire curve, which is a very rare case. We are more often concerned with predicting short-term rate shifts and considering their consequences for the rest of the yield curve, and thus also for bonds with longer maturities. And this is why it is useful to find a certain measure that represents the impact of short rate shifts on bond prices with respect to bond parameters.

Thus, the main contribution of this financial engineering study is to propose a measure that can be used in the same way as Macaulay duration, but not only as a response to a change of YTM, but as a response to a change of the short interest rate. By "use" we mean, for example, its usage in equation for changing ΔP of a bond, in the equation of the volatility ratio of two bonds, or in the equation for the interest rates sensitivity of the bond portfolio. Such a measure is still lacking in finance. We refer to this measure as "short rate shift duration". Since the effect of short-rate shift on the entire yield curve, and thus on the price of long-term bonds in particular, is very

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difficult to predict analytically, we use empirical data to calculate the short-rate shift duration value and we also calculate its values for the USD and EUR interest rate markets.

Mainly the movements of the very short-term interest rates, as dictated by central banks, affect bonds with different maturities in different way, depending also on the market expectations of future levels of inflation. For example, a change in short-term interest rates that do not affect long end of zero-coupon curve will have little effect on a long-term bond price and its YTM. However, a change in short-term interest rates that affect long-term interest rates can greatly affect a long-term bond price and its YTM. Put simply, changes in short-term interest rates have a more clear effect on short-term bonds, as the short-term bond YTM usually follows the short-term rate shift direction. On the other hand, the effect on long bonds is more difficult to predict. Sometimes the long-term zero-coupon yields go in the same direction with the short-term ones, sometimes the zero-coupon curve goes inverse.

There are many potential areas for the application of our approach. One example being one certain bond life, during which its term to maturity is decreasing and the price volatility is changing while short interest rate shifts influence the whole zero-coupon curve; another application is a straightforward extension, that being a portfolio of bonds with different maturities at a certain point of time. The price of an asset in the light of a change in interest rates and its measurement of price sensitivity is a crucial question for assessing market risk in the case of a portfolio of interest-rate sensitive assets, such as bonds or loans (Stádník, 2022).

The limitation of the research is that we do not study dependence of sensitivity on credit risk, but we use zero-coupon rates constructed using government risk investment instruments. The results of this research are confirmed in the diploma thesis (Havlíková, 2021), in which the basic ideas of short rate shift duration were used and are also supplemented by the dependence of the short rate shift duration on the credit rating. It is clear from the results that the values of the short rate shift duration in the case of a deteriorating rating depend less on the term to maturity.

This research is typical one of financial engineering. We also provide certain examples in the text in order to explain the strategy better to financial practitioners and to show its difference from risk management techniques (Janda & Kourilek, 2020).

1. Literature review

The concept of a short rate shift duration is quite close to the concept of other durations, which basically address the sensitivity of the bond price with respect to more parameters than just the coupon rate, the term to maturity, the face value of the bond and the change of YTM. One of these durations that should be mentioned is empirical duration. But the definition of empirical duration is often very vague, sometimes it is defined in the sense of modified duration (Čerović et al., 2014), and usually, it is defined using comparison of empirical changes of prices of corporate bonds with respect to the yield changes of government bonds.

Fooladi et al. (1997) define duration for bonds, which does not ignore default risk, lead to error in the two major duration applications (according to their statement) – measuring interest rate price elasticity and immunization. They derive a general expression for duration in the presence of default risk based on Jonkhart's term structure model (Jonkhart, 1979) extended to encompass risk aversion. The model includes terms for default probabilities and default payoffs in each period, as well as for a delay between the occurrence of default and the final default payoff. Their main conclusion is that practical duration applications, involving bonds with default risk, must employ duration measures adjusted for default risk.

In our study, we use short rate shift as a key factor for changing of shape of the whole zero-coupon curve. Behavior of short-term interest rate was described, for example, by Haitao and Yuewu (2009). They characterize the dynamics of the US short-term interest rate using a Markov regime-switching model and show that there are two regimes in the data: In one regime, the short rate behaves like a random walk with low volatility; in another regime, it exhibits strong mean reversion and high volatility. In their model, the sensitivity of interest rate volatility to the level of interest rate is much lower than what is commonly found in the literature.

Our research is important for determination of bond interest rate sensitivity and, of course, for resulting volatility of bond. Volatility issues have an important impact on risk management of a bond portfolio. Important works in the field of bonds volatility are Litterman et al. (1991) and volatility of bonds' determinants (Fuller & Settle, 1984). Serious research in the area of volatility is also provided in Stádník (2012, 2014), Stádník and Žďárek (2017) (version for practical portfolio management) which belong to a strand in the financial literature focused on basic bonds behaviour such as Chance and Jordan (1966), Litterman and Scheinkman (1991), Fabozzi (1993, 1995, 2010) or Smit and Swart (2006).

The research (Stádník, 2012) introduced a definition of three different regimes of common bond clean price volatility development and examines the theoretical and practical repercussions of such phenomena as an extension to the existing literature. A way of determining values of switching points (interest rate values) between these regimes with respect to the level of interest rates using numerical calculations are presented and explained. The text includes numerical solving for switching points for maturities from 1 up to 1200 years that show that the switching point 1 (between the regime of the "typical" development with decreasing volatility and other regimes) is of lower value for higher maturities, which is also in accordance with Fuller and Settle (1984). We can also state that for higher maturities the switching point has its practical value within the meaning of today's low levels of interest rates. The switching point

2 (starting point of "inverse" volatility development) is not of value less than 50%. Switching points decide the shape of the volatility envelope and if the clean price of a bond is developing inside the volatility envelopes, its sensitivity (volatility) increases / decreases according to the shown shape of the envelope.

Other interesting research in the field of bonds behavior, their risks and valuation is made by Kang and Chen (2002) or Křepelová and Jablonský (2013). They carried out studies on the development of prices in the bond market in the medium- and long-term periods, which are linked to the specificity of these particular periods of time. We should also mention research concerning valuation of interest rate sensitive assets like Tvaronavičienė and Michailova (2006) and Visokavičienė (2008), behaviour of a bond portfolio in Dzikevičius and Vetrov (2013), research in bond portfolio immunization by Ortobelli and Tichý (2015), Giacometti et al. (2015) or Ortobelli et al. (2018). Steeley (2006) made the study which concerns the transfer of volatility between stocks and bonds. Brůna and Blahová (2016) made the study concerning liquidity issues and Webb (2015) solved an interesting problem in the area of negative interest rates. Ho (1992) discusses about "key rate duration", which is defined as the sensitivity of the bond or of the bond portfolio to the given key rates only at a certain point along the zero-coupon term structure. But its practical significance is apparently not significant.

2. Methodology

2.1. Basic theory for methodology

Since mathematically short rate shift duration should be analogical to Macaulay Duration, we will proceed from standard relationships.

$$P(YTM)=\left(1+YTM\frac{l}{T}\right)^{-1}\times \left[c+\frac{c}{(1+YTM)}+\frac{c}{(1+YTM)^2}+\ldots+\frac{c+100}{(1+YTM)^{n-1}}\right], \quad (1)$$

where $P(YTM)$ is the dirty price of a bond determined in the percentage of its face value on purchasing day, c is the coupon rate per the coupon period, YTM is the yield to maturity (determined per the coupon period and uses the compounding period of the same length), l is the number of days till the next coupon payment, n is the number of coupon payments till the maturity and T is the number of days inside the coupon period. l, T depends on the day count convention. For the ex-coupon day (if the settlement of the coupon is on this day) we may use Eq. (2a):

$$P(YTM)=\frac{c}{(1+YTM)}+\frac{c}{(1+YTM)^2}+\ldots+\frac{c+100}{(1+YTM)^n}. \quad (2a)$$

We may consider the total price also to be the sum of total prices of n zero-coupon bonds:

$$P(i_1,i_2,\ldots i_{mat})=\frac{c}{(1+i_1)}+\frac{c}{(1+i_2)^2}+\ldots+\frac{c+100}{(1+i_{mat})^n}, \quad (2b)$$

where i_1, i_2,…i_{mat} are zero-coupon rates for maturities 1,2,….n years, and all other variables have the same interpretation as in the Eq. (1).

Based on the Taylor's theorem for a real-valued function f differentiable at the point a, there is a polynomial approximation of a higher degree (quadratic, cubic, quartic…) at the fixed point a. Taylor's theorem provides this approximation in a sufficiently small neighbourhood (h) of the fixed-point a:

$$f(a+h)=f(a)+f'(a)h+\frac{f''(a)h^2}{2!}+\frac{f'''(a)h^3}{3!}+\ldots+R. \quad (3)$$

If Equation (3) is applied to Eq. (2a) and with the substitution: h for ΔYTM and $f(a)$ for P, consequently, it can be shown that:

$$\Delta P(YTM)=P'(YTM)\Delta YTM+\frac{P''(YTM)\Delta YTM^2}{2!}+\frac{P'''(YTM)\Delta YTM^3}{3!}+\ldots+R, \quad (4)$$

where R is the remainder of the series and ΔYTM is the change in the market interest rate. Using the Eq. (4) for ΔP (as percentage of its face value) as the general measure of volatility and only up to the second order approximation:

$$\Delta P(YTM)\cong -DUR_{MAC}\frac{P(YTM)}{(1+YTM)}\Delta YTM+\frac{1}{2}P(YTM)CONV\,\Delta YTM^2, \quad (5)$$

where DUR_{MAC} is the Macaulay duration and the term $CONV$ stands for the convexity of the bond. Macaulay duration, utilizing Eq. (4), could be determined in this way:

$$DUR_{MAC}=\frac{\sum_{k=1}^{n}\left[\frac{kc}{(1+YTM)^k}\right]+\frac{100n}{(1+YTM)^n}}{P}. \quad (6)$$

2.2. Short rate shift duration definiton

First, let us define short rate shift duration on daily basis DUR_{SRSd} using the first term of Eq. (5) in the following way:

$$DUR_{SRS_d}\cong -\frac{(1+YTM_d)}{P_d}\frac{\Delta P_d}{\Delta i_1}, \quad (7)$$

where the parameters are described in the Table 1.

Table 1. Parameters legend

DUR_{SRSd}	short rate shift duration on daily basis on day *d*
YTM_d	yield to maturity of bond on day *d*
Δi_1	change of the short-term interest rate (1-year) between *d* and *d-1*, calculated based on reshaping of yield curve in Figure 1
P_d	price of bond with respect to its time to maturity and the structure of zero-coupon curve on day *d*
ΔP_d	change of the price of a bond between *d* and *d-1*, calculated based on reshaping of yield curve in Figure 1
d	number of days (from the beginning of time series, Figure 1)

In comparison to Macaulay duration, in the case of valuation of short rate shift duration, we use 1 year maturity interest rate change for all the maturities.

Let us define short rate shift duration as the mean value of daily short rate shift durations:

$$DUR_{SRS} = E\left[DUR_{SRS_d}\right], \quad (8)$$

where mean value is denoted as E[X],where X is vector to which components we apply the average.

2.3. Short rate shift duration of portfolio

With short rate shift duration, we may deal in the same way as with Macaulay duration and also express short rate shift duration of portfolio using analogical equation as it is in the case of Macaulay duration:

$$DUR_{SRS\_PORTF} \cong \sum_{j=1}^{n} w_j DUR_{SRS\,j}, \quad (9)$$

where w_j is the weight of j^{th} asset in the portfolio.

2.4. Methodology steps

The methodology comprises into the following steps:

1. Based on empirical data, we create a graph of the development of the shape of USD and EUR zero-coupon curves (Figure 1 and Figure 6) day after day (on a daily basis).
2. Based on the zero-coupon curves rates and using Equation (2b), we calculate the price P_d and adequate daily change of price ΔP_d of fixed coupon bonds with maturities 1 to 30 years and coupons from 1 to 7%. Frequency of coupon payment =1 year, no embedded option. We speak about typical coupon bond. Credit risk is the same as the risk of financial instruments that were used to construct the yield curves in the Figures 1 and 6, which is government risk.
3. Based on the knowledge of price, we determine YTM_d from Eq. (1).
4. Based on the daily price changes, we calculate adequate daily short rate shift duration using Eq. (7). Note that instead of ΔYTM for each maturity we use change of 1-year zero-coupon rate Δi_1.
5. We calculate short rate shift duration DUR_{SRS} as mean of daily short rate shift durations DUR_{SRSd}, see Eq. (8).
6. We calculate values of Macaulay duration for different maturities and compare to the values of short rate shift duration, for demonstration, for coupon equals 5%.
7. We calculate the values of short rate shift duration for different coupons.
8. We calculate the values of short rate shift duration for zero-coupon bonds.
9. We use polynomial fitting to construct a formula for calculations of short-term shift duration for USD with respect to different coupons and maturities.

3. Results

3.1. USD zero-coupon rate case

We may observe many cases of inverse interest rate shifts along zero-coupon yield curve which could decrease the change of price in a case of a typical coupon bond in comparison to our estimation using Macaulay duration, which use the same shift along the whole curve. By inverse shift we mean when the shape of the yield curve changes so that the yield on short and long maturities moves in the opposite direction.

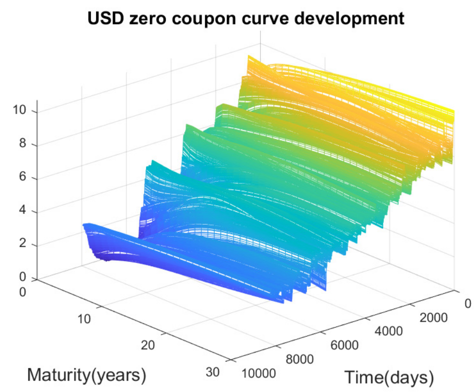

Figure 1. Daily shape changes of USD zero-coupon yield curve, period 1999–2018 (4900 working days) (source: Reuters)

Average day to day ratio of price shifts (1–30 years over 1 year maturity) based on the development of the shape of zero-coupon curve in the Figure 1 is displayed in the Figure 2. According to Macaulay duration, the ratio of price changes should be approximately equaled to the ratio of Macaulay durations. Figure 2 shows that the ratio of shifts using empirical data is smaller, starting with maturities of 5 years or more, than would correspond to estimates using Macaulay duration.

Ratio of inverse bond price shifts of different maturities with respect to maturity equaled 1 year is in the Figure 3a.

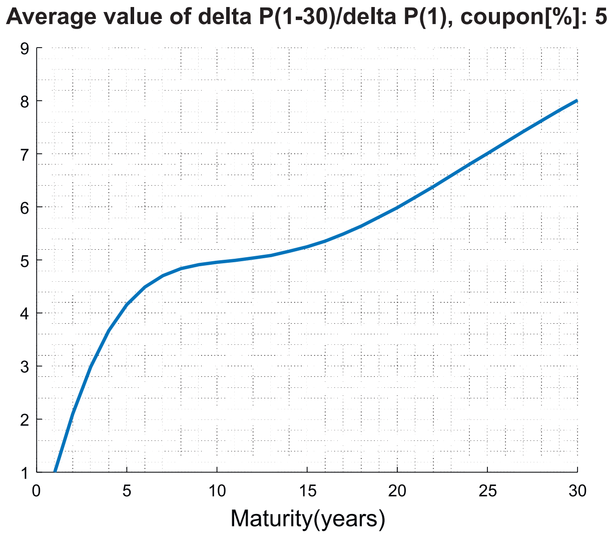

Figure 2. Average value of ratio of bond prices shifts

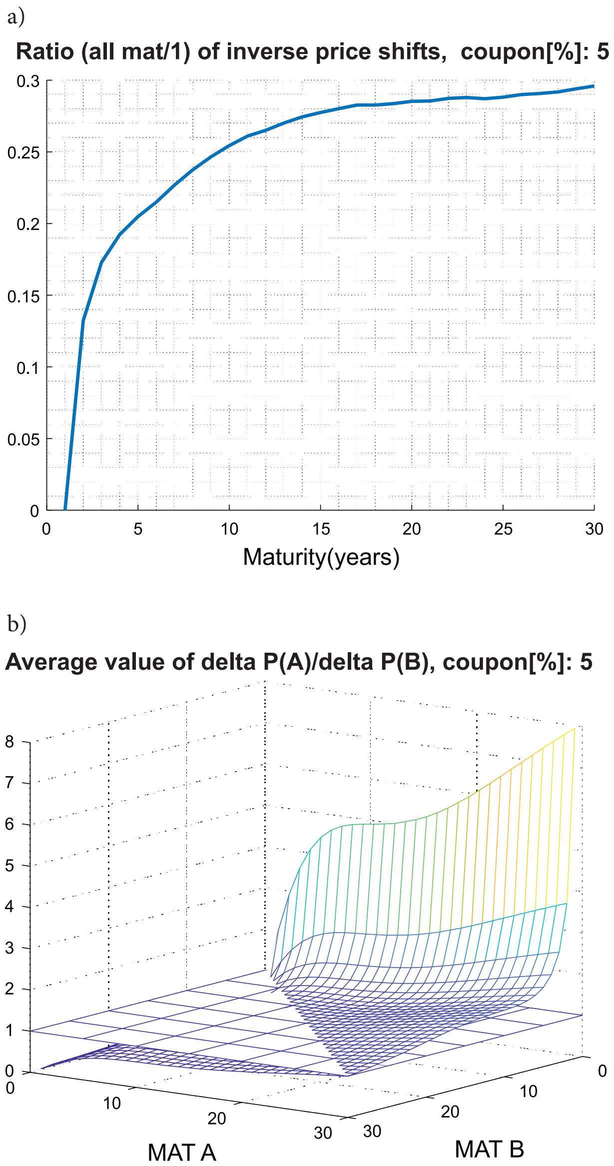

Figure 3. Ratio of inverse bond price shifts of different maturities with respect to maturity equaled 1 year (a); ratio of bond price shifts with respect maturities to each other (b)

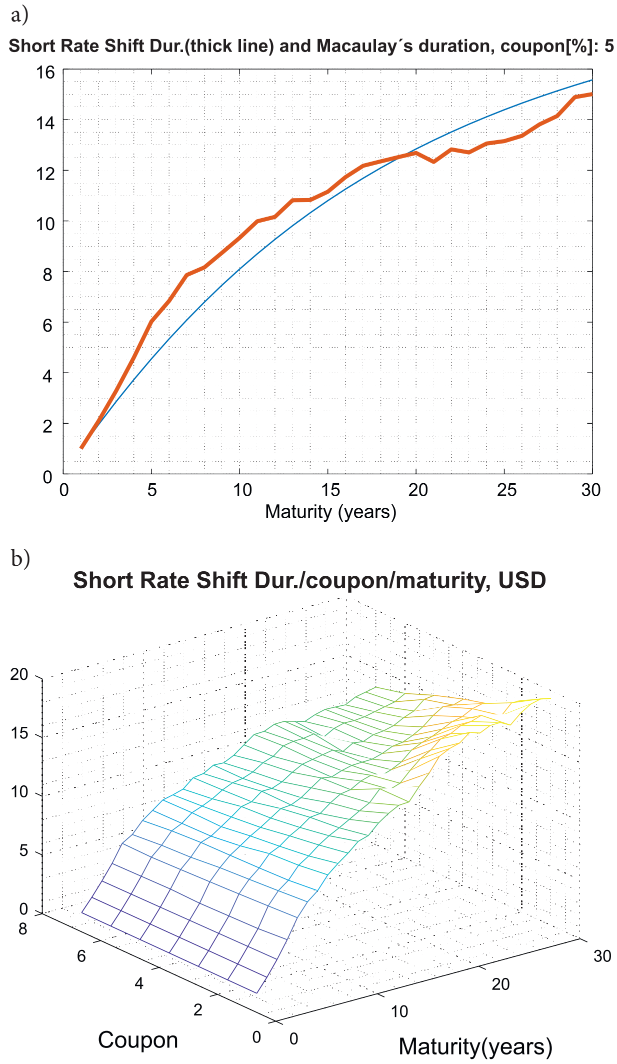

Figure 4. Short rate shift duration (thick line) and Macaulay duration, coupon = 5% (a); short rate shift duration with respect to coupon rate and maturity (b)

From the figure, it is clear that approximately in 25–30% cases, the price of long fixed coupon bond (longer than 15 years) is forced in the opposite direction than the price movement of 1 year bond.

Ratio of bond price shifts with respect maturities to each other is in the Figure 3b. Values of short rate-shift duration for bonds with coupon 5% with respect to maturity are compared with the Macaulay duration values in Figure 4a. The values of short rate shift duration are shown by a thick line in the Figure. It is interesting to mention that the value may be higher than the time to maturity of the bond, which in the case of Macaulay duration cannot occur. Figure 4b shows the values of short rate shift duration for coupons 1–7% p.a.

With higher coupon the value of short rate shift duration is lower. The same feature we observe in the case of Macaulay duration. Exact values are in the Table 2.

For zero-coupon rate the value of short rate shift duration is in the Figure 5 (thick line). Important is that its values are not the same as maturity as it is in the case of Macaulay duration.

Table 2. Short rate shift duration, USD

MAT\ COU-PON	1	2	3	4	5	6	7
1	1,00	1,00	1,00	1,00	1,00	1,00	1,00
2	2,13	2,12	2,11	2,10	2,09	2,08	2,07
3	3,42	3,38	3,35	3,32	3,29	3,26	3,23
4	4,86	4,67	4,60	4,54	4,60	4,54	4,49
5	6,50	6,36	6,24	6,13	6,03	5,93	5,97
6	7,61	7,42	7,25	7,10	6,84	6,72	6,48
7	8,34	8,08	7,99	7,91	7,86	7,71	7,44
8	9,45	9,12	8,97	8,49	8,16	8,11	8,07
9	10,19	9,68	9,36	9,09	8,74	8,78	8,72
10	10,83	10,38	10,01	9,59	9,33	9,11	8,92
11	11,60	10,93	10,50	10,15	9,99	9,50	9,18
12	11,86	11,38	11,12	10,72	10,16	10,02	9,92
13	12,70	11,86	11,21	10,80	10,82	10,40	10,04
14	13,29	12,35	11,88	11,18	10,83	10,53	10,53
15	13,74	12,81	12,41	11,79	11,15	10,85	10,48
16	13,78	13,76	12,55	12,39	11,73	11,40	10,89
17	14,86	13,47	13,04	12,24	12,18	11,71	11,54
18	15,79	13,77	13,30	12,55	12,35	12,37	12,19
19	16,97	14,64	13,02	12,76	12,52	12,52	12,23
20	17,40	15,38	13,30	12,80	12,69	12,54	12,60
21	17,64	15,87	13,73	12,82	12,33	12,69	12,61
22	18,31	16,60	14,50	13,08	12,83	12,20	12,38
23	18,02	16,83	14,90	13,45	12,71	12,68	12,13
24	18,42	17,28	15,18	13,56	13,06	12,91	12,48
25	18,44	17,38	15,58	13,92	13,15	12,88	12,69
26	17,99	17,67	15,98	14,40	13,36	12,97	12,66
27	18,76	17,08	16,18	15,13	13,82	13,17	12,61
28	19,31	17,18	16,62	15,66	14,15	13,36	13,05
29	19,52	18,04	16,83	15,56	14,89	13,55	13,23
30	19,33	18,00	16,50	15,83	15,01	14,02	13,41

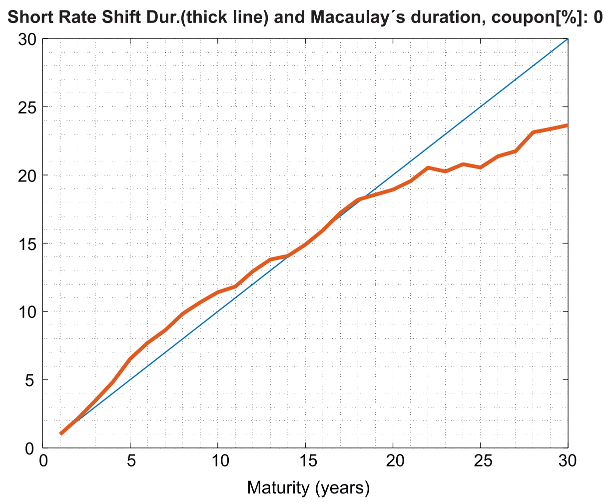

Figure 5. Short rate shift duration (thick line) and Macaulay duration (thin line) for zero-coupon bond

3.2 The short rate shift duration for EUR market

In the case of the EUR market, we will proceed in an analogous way as in the case of the USD market and will therefore use the same methodology as in the previous chapter.

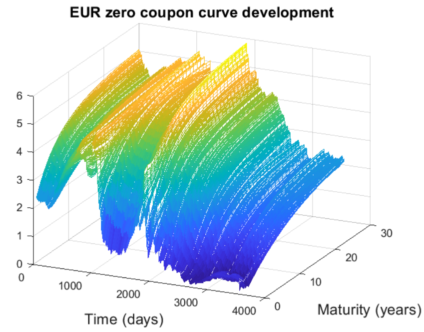

Figure 6. Daily shape changes of EUR zero-coupon yield curve, period 2004–2018 (3500 working days) (source: Reuters)

In case of EUR market, we also empirically observe interest rate shift inverse movements (between long- and short-term maturities), which could decrease the change of bond price in a case of a typical coupon bond in comparison to our estimation using Macaulay duration, which uses the same shift along the whole zero-coupon curve.

Average day to day ratio of price shifts (1–30 years over 1 year maturity) based on the development of the shape of zero-coupon curve in the Figure 6 is displayed in the Figure 7.

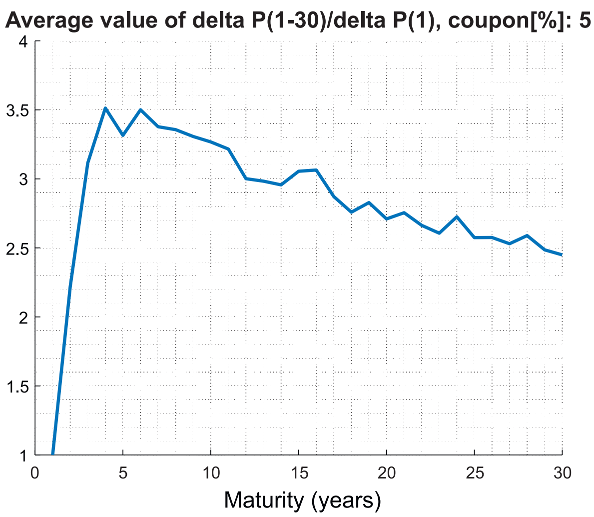

Figure 7. Average value of ratio of bond prices shifts

Figure 7 shows that the ratio of shifts using empirical data is smaller, starting with maturities of 3.5 years or more, than would correspond to estimates using Macaulay duration.

Ratio of inverse bond price shifts of different maturities with respect to maturity equaled 1 year is in the Figure 8a. From the Figure, it is clear that approximately in 35–40% cases, the price of long bond (longer than 15 years) is forced in the opposite direction than the price movement of 1 year bond.

a)

b)

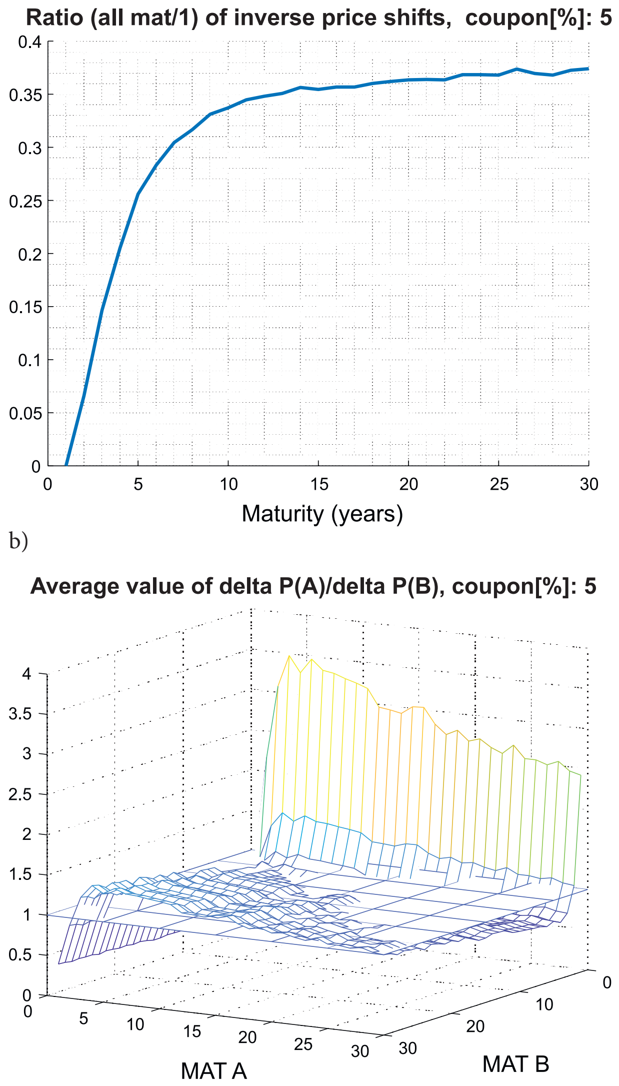

Figure 8. Ratio of inverse bond price shifts with respect to maturity equaled 1 year (a); maturities with respect to each other (b)

Ratio of bond price shifts with respect maturities to each other is in the Figure 8b. Values of short rate shift duration in comparison to Macaulay duration are in the Figure 9a (thick line in the Figure) for coupon 5%, Figure 9b for coupons 1–7% p.a.

Exact values are in the Table 3.

Table 3. Short rate shift duration, EUR

MAT\ COU-PON	1	2	3	4	5	6	7
1	1,00	1,00	1,00	1,00	1,00	1,00	1,00
2	2,21	2,20	2,19	2,18	2,17	2,16	2,15
3	3,10	3,07	3,04	3,02	2,99	2,97	2,95
4	3,51	3,47	3,43	3,39	3,36	3,33	3,30
5	3,36	3,32	3,28	3,25	3,22	3,20	3,17
6	3,61	3,56	3,51	3,47	3,43	3,40	3,37
7	3,57	3,51	3,46	3,42	3,39	3,36	3,33
8	3,62	3,56	3,51	3,47	3,43	3,40	3,37
9	3,68	3,61	3,55	3,50	3,46	3,43	3,40
10	3,73	3,65	3,59	3,54	3,49	3,46	3,43
11	3,77	3,68	3,61	3,56	3,51	3,47	3,44
12	3,63	3,55	3,49	3,44	3,41	3,38	3,35
13	3,71	3,62	3,55	3,49	3,45	3,42	3,39
14	3,82	3,70	3,62	3,56	3,51	3,47	3,44
15	4,09	3,93	3,82	3,74	3,67	3,62	3,58
16	4,25	4,06	3,93	3,83	3,76	3,70	3,65
17	4,12	3,94	3,82	3,73	3,67	3,62	3,58
18	4,12	3,93	3,81	3,72	3,66	3,61	3,57
19	4,42	4,16	4,00	3,89	3,81	3,74	3,70
20	4,38	4,13	3,96	3,86	3,78	3,72	3,67
21	4,67	4,34	4,14	4,01	3,91	3,84	3,78
22	4,69	4,35	4,14	4,01	3,91	3,84	3,79

a)

b)

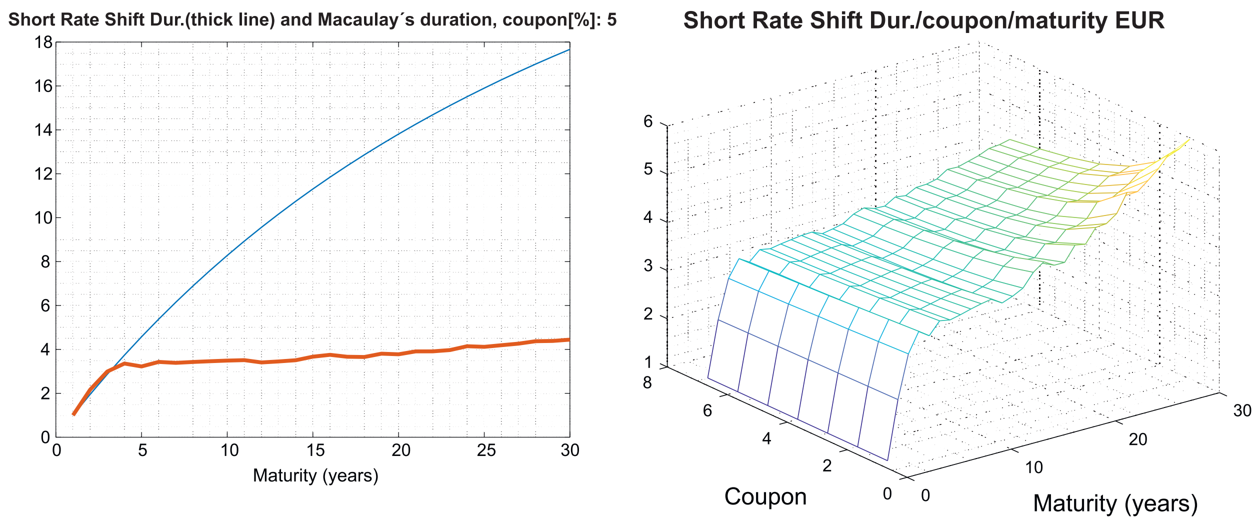

Figure 9. Short rate shift duration (thick line) and Macaulay duration (thin line), coupon = 5% (a); short rate shift duration with respect to coupon rate and maturity (b)

End of Table 3

MAT\ COU-PON	1	2	3	4	5	6	7
23	4,84	4,45	4,22	4,07	3,97	3,90	3,84
24	5,22	4,73	4,45	4,27	4,15	4,05	3,98
25	5,18	4,69	4,42	4,24	4,12	4,03	3,97
26	5,35	4,81	4,51	4,32	4,19	4,10	4,03
27	5,54	4,93	4,60	4,40	4,26	4,16	4,09
28	5,78	5,11	4,75	4,52	4,37	4,27	4,18
29	5,83	5,13	4,76	4,54	4,39	4,28	4,20
30	5,97	5,23	4,84	4,60	4,45	4,34	4,25

For zero-coupon bond, the value of short rate shift duration is in the Figure 10. Important is that its values are not the same as maturity as it is in the case of Macaulay duration.

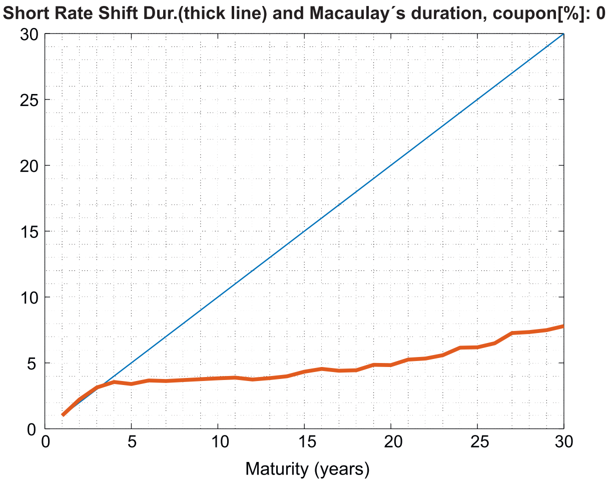

Figure 10. Short rate shift duration (thick line) and Macaulay duration for zero-coupon bond

3.3. Polyfit ( ) in Case of USD

As a final step, we tried to put together a formula for calculating the values of short rate shift duration depending on the coupon and term to maturity, for the USD market.

To find the mathematical formula to express short rate shift duration with respect to maturity and coupon we use Matlab function P = **polyfit** (X, Y, N). P is polynomial function of degree N that fits the data Y best in a least-squares sense. Y is represented by the values of short rate shift duration in the Table 2. We use degree $N = 2$ for fitting the maturity and $N = 3$ for fitting the coupon. Resulting formula is:

$$DUR_{SRS} = \sum_{j=1}^{3}[(k_{(j,1)}coupon^3 + k_{(j,2)}coupon^2 + k_{(j,3)}coupon^1 + k_{(j,4)})\Delta mat^{3-j}], \quad (10)$$

where *coupon* are values of coupon rate, *mat* is maturity of a bond and coefficients k are in the Table 4.

Table 4. k (row, column)

row/column	1	2	3	4
1	0.0001	–0.0014	0.0065	–0.0254
2	–0.0024	0.0458	–0.2790	1.4777
3	0.0077	–0.1636	0.9900	–0.6195

The plane in the Figure 11 is constructed using Eq. (10) and it should correspond to the plane in the Figure 4. It can be said that the Eq. (10) can be used.

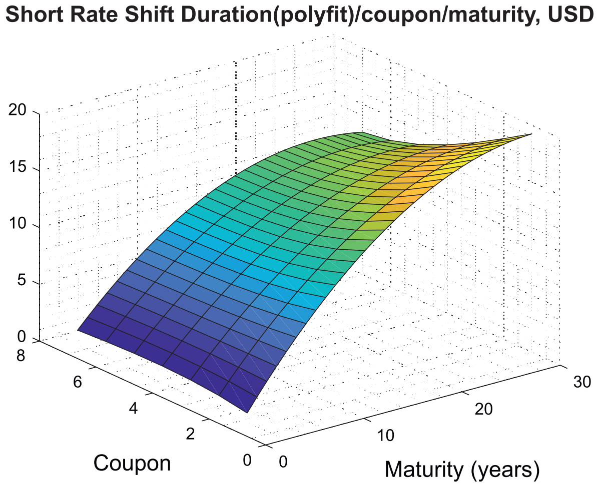

Figure 11. Short rate shift duration plane resulting from mathematical formula/coupon/maturity

Conclusions

In this financial engineering research, we have defined a new duration – the short rate shift duration, which is based on measuring the sensitivity of the bond price with respect to the change in the value of the short-term interest rate. In the case of our research – one year. In our opinion, the short rate shift duration has more practical significance than Macaulay duration, which deals with the response of the bond price to the change of yield to maturity. A change of yield to maturity is a much more complex parameter than a change of a short interest rate, and most of the time, because of the difficulty of assessing it, we do not even try to estimate its future values.

Short rate shift duration is a certain measure that can be handled in the same way as conventional Macaulay duration, for example: in the equation for changing the ΔP of the bond, for the volatility ratio of two bonds, or in the equation for the bond portfolio duration. Such a measure is still lacking in finance.

A certain disadvantage compared to Macaulay duration is the fact that the values of the short rate shift duration must be calculated numerically and separately for individual segment of the bond market because the effect of short rate shift on the entire yield curve, and thus on the price of long-term bonds in particular, is very difficult to predict analytically. In our research, we calculated its values using the USD and EUR zero-coupon curve for typical fixed coupon bonds and with government risk. In the text above, however, we mention a reference (Havlíková, 2021) to a follow-up work that also considers different credit ratings. We also used polynomial fitting to construct a formula for calculations of short-term shift duration for USD with respect to different coupons and maturities. This formula was constructed more for demonstration and then verified using empirical data.

Since the movements of the zero-coupon curve shape are inverse (the long and short ends of the curve move in the opposite direction) in approximately 30% of all cases, we expect the price volatility of long-maturity fixed coupon bonds to be lower than it should be according to Macaulay duration, which means that we can conclude that long bonds are not as risky as they are thought to be. The short rate shift duration, which essentially includes the quantification of such an effect, supported these expectations.

The value of the short rate shift duration also decreases with a higher coupon and increases with a longer maturity as in the case of the Macaulay duration. In the case of a zero-coupon bond, the value of a short rate shift is not the same as the maturity period, as is the case with Macaulay duration.

Funding

This research has been supported by the VSE institutional grant IP 100040.

Compliance with Ethical Standards

I do not have any competing financial, professional, or personal interests from other parties. This article does not contain any studies with human participants or animals performed by any of the authors.

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